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Floquet instability of a gravity modulated Rayleigh–Benard problem in an anisotropic porous medium

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A R T I C L E I N F O

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A B S T R A C T

This work deals with the effect of gravity modulation on the onset of convection in a horizontal porous layer subjected to an adverse temperature gradient. Low amplitude gravity modulations are considered. The analysis is linear and specific attention is paid to the boundary effects of Brinkman's model and anisotropies of the porous medium in permeability and thermal conductivity. A Floquet analysis is applied and critical values of the parameters are found analytically using stability charts. The emergence of instability through the synchronous and subharmonic modes, transition between them and their dependence discussed for physically realistic values of control parameters. The findings of this analysis may be useful in controlling convective plumes during fabrication which develop into freckles in the ground grown crystals.

1. Introduction

The knowledge of thermal convection in mushy layers of mixed solid and liquid phases has become increasingly important during the fabrication of single crystals required by the present day technology. There are significant differences in the compositional homogeneity and structural perfection between space grown and ground grown crystals. The unfavourable buoyancy driven convection, sedimentation and hydrostatic pressure in the process of crystal growth can be suppressed under microgravity environment. A knowledge of these allows us to understand the reason behind crystal growth related defect formation and determine the best way to produce high quality crystals on the ground. Several ways are being adopted to have a control on the buoyancy driven flow like reduced gravity level, rotation, agitation techniques, etc. The growing volume of work devoted to this area is well documented by the most recent review of Razi et al. [1].

The Rayleigh–Benard problem in a porous domain with variable gravity effects has been studied by many researchers (see for example Alex and Patil [2] and Saravanan and Kandaswamy [3]). Time-dependant body forces may occur in systems, with density gradients, subjected to vibrations. The influence on thermal convection depends on the orientation of the fluctuating body force with respect to the thermal stratification. This type of body force can even alter the stable distribution of a stratifying agency under constant gravity environment and introduce parametric resonance under suitable conditions. Much work has been done in vertically modulated pure fluid layers with constant vertical stratification, i.e., modulated Rayleigh–Benard convection. This type of modulation in gravity may be realized by vertically oscillating a fluid layer in a constant gravitational field. Gershuni et al. [4] and Gresho and Sani [5] were the first to initiate the study of the effect of gravity modulation in a fluid layer. But the study of this aspect in a porous medium is comparatively of recent origin.

Malashetty and Padmavathi [6] asymptotically analyzed the linear stability of a horizontal fluid saturated porous layer heated from below for the case of low amplitude gravity modulation. They found that the low frequency gravity modulation significantly affects the system for both Darcy and Brinkman models. Bardan and Mojtabi [7] studied numerically and analytically convection in a rectangular fluid saturated porous cavity heated from below and subjected to high frequency vibration. They found that increasing the vibration amplitude increases critical Rayleigh number to large values and may even create subcritical solutions. Recently Govender [8,9] has made stability analyses to investigate the effect of low amplitude gravity modulation on convection in a homogeneous porous layer heated from below. By plotting Mathieu's stability charts he could predict the actual transition point from synchronous to subharmonic mode. The onset of thermocapillary convection in a fluid saturated porous medium subjected to vertical

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Nomenclature

b_*	vibration amplitude
Da	Darcy number, K_{z*}/H_*^2
Fr	Froude number, $\lambda_{z*}^2/(\overline{g}_* H_*^3)$
$\overline{g}_*$	gravitational acceleration
H_*	the height of the porous layer
$\overline{\overline{K}}_*$	permeability tensor, $K_{x^*}(\overline{i}\,\overline{i}+\overline{j}\,\overline{j})+K_{z^*}(\overline{k}\,\overline{k})$
L_*	the length of the porous layer
p	pressure
Pr	Prandtl number, ν_*/λ_{z*}
R	Rayleigh number, $\beta_*\Delta T_C g_* K_{z*} H_*/\nu_*\lambda_{z*}$
R_v	vibrational Rayleigh number
s	wavenumber
T	temperature, $(T_* - T_C)/(T_H - T_C)$
t_*	time
T_C	cold wall temperature
T_H	hot wall temperature
u	horizontal x component of the filtration velocity
v	horizontal y component of the filtration velocity
$\overline{V}$	filtration velocity vector, $u\overline{i}+v\overline{j}+w\overline{k}$
$\overline{V}_1$	anisotropy modified velocity vector, $(u/\xi)\overline{i}+(v/\xi)\overline{j}+w\overline{k}$
Va	Vadasz number, $\varphi Pr/Da$
w	vertical z component of the filtration velocity
W_*	the width of the layer
x	horizontal length coordinate
y	horizontal width coordinate
z	vertical coordinate

Greek symbols

α	a parameter related to the wavenumber, s^2/π^2
β_*	thermal expansion coefficient
γ	Va/π^2
δ	$\kappa Fr \Omega^2$
ΔT_C	characteristic temperature difference
κ	b_*/H_*
$\overline{\overline{\lambda}}_*$	thermal diffusivity tensor, $\lambda_{x^*}(\overline{i}\,\overline{i}+\overline{j}\,\overline{j})+\lambda_{z^*}\overline{k}\,\overline{k}$
μ_*	dynamic viscosity
μ_{e^*}	effective dynamic viscosity
$\overline{\mu}$	μ_{e*}/μ_*
ν_*	kinematic viscosity
ξ	mechanical anisotropy parameter, K_{x*}/K_{z*}
η	thermal anisotropy parameter, $\lambda_{x*}/\lambda_{z*}$
ς	scaled exponent, equals $\sigma/\sqrt{-a}$
ρ	fluid density
σ	Mathieu exponent
ϕ	porosity
ω_*	vibration frequency
Ω	scaled vibration frequency, $\omega_* H_*^2/\lambda_{z*}$

Subscripts

$*$	dimensional quantity
c	characteristic
cr	critical
C	cold wall
H	hot wall
o	unmodulated quantity

vibration was examined by Zenkovskaya [10] using the averaging method and it was concluded that high frequency vibration stabilizes as well as destabilizes the no-flow state. He found that the horizontal vibration destabilizes the system and augments convection in zero gravity and microgravity. Also he concluded that the effect of modulation disappears for high frequencies.

The study of convection in porous media of anisotropic nature has become crucial due to its inherent occurrence in natural and artificial porous structures. The non-uniformities may be attributed to either changes in permeability or thermal conductivity or both of the porous matrix. Most of the earlier studies have used the classical Darcy's law which is valid for flow through regular structures over the whole spectrum of the porosity. Hence this model is silent about the flow structure near the bounding surfaces. Malashetty et al. [11] studied the effect of thermal modulation on the onset of convection in a horizontal, anisotropic porous layer saturated with a viscoelastic fluid. Darcy's law with viscoelastic correction was used to describe the fluid motion. The stability of the system characterized by a Rayleigh number was calculated as a function of anisotropy parameters, viscoelastic parameters and frequency of modulation. Govender [12] investigated natural convection in an anisotropic porous layer subjected to centrifugal body forces. He found that the convection is stabilized when the thermal anisotropy ratio, which is a function of mechanical and thermal anisotropy parameters, is increased in magnitude.

The objective of the current work is to extend the use of Mathieu's stability charts for studying the onset of modulated Rayleigh–Benard convection in a more general porous medium. We employ Brinkman's equation to model the momentum balance. It is appropriate for a highly porous medium and when the Darcy number is not small and takes care of the boundary effects. In addition the porous domain is assumed to be anisotropic in mechanical as well as thermal sense. In particular we focus on a transversely anisotropic medium in which the principal axes of permeability are aligned with the coordinate frame.

2. Problem formulation

A shallow and sparsely packed anisotropic horizontal fluid saturated porous layer confined between the surfaces $z_* = 0$ and $z_* = H^*$ is considered. The layer is heated from below and is subjected to vibration in a direction parallel to the gravitational field. Brinkman's law is used and the convective terms are neglected as we deal with a quiescent initial state. The equations governing the above system under the assumption of the Boussinesq approximation are

$$\nabla_* \cdot \overline{V}_* = 0 \tag{1}$$

$$\frac{\rho_{c*}}{\phi}\frac{\partial \overline{V}_*}{\partial t_*} = -\nabla_* p_* - \frac{\mu_*}{\overline{\overline{K}}_*}\overline{V}_* + \mu_{e^*}\nabla_*^2\overline{V}_* - (\rho_* - \rho_{c*}) \times \left(\overline{g}_* + b_*\omega_*^2 \sin(\omega_* t_*)\right)\overline{k} \tag{2}$$

$$\frac{\partial T_*}{\partial t_*} + \overline{V}_* \cdot \nabla_* T_* = \nabla_* \cdot (\overline{\overline{\lambda}}_* \cdot \nabla_* T_*). \tag{3}$$

where $\overline{\overline{K}}_*$, $\overline{\overline{\lambda}}_*$, $\overline{V}$, T, p and ϕ represent the permeability, thermal diffusivity, filtration velocity, temperature, reduced pressure and porosity, respectively. We use the following transformation

$$(u_*, v_*, w_*) = \frac{\lambda_{z*}}{H_*}(u, v, w), \quad p_* = (\mu_*\lambda_{z*}/K_{z*})p,$$

$$(T_* - T_C) = \Delta T_c T = (T_H - T_C)T, \quad (x_*, y_*, z_*) = H_*(x, y, z),$$

$$t_* = \frac{H_*^2}{\lambda_{z*}}(t), \quad \rho_* = \rho_{c*}(1 - \beta_*\Delta T_c T),$$

$\mu_* = \nu_* \rho_{c*}, \quad R = \beta_* \Delta T_c g_* K_{z*} H_* / \nu_* \lambda_z.$

and express Eqs. (1)–(3) in non-dimensional form as

$$\nabla \cdot \overline{V} = 0 \tag{4}$$

$$\frac{1}{Va}\frac{\partial \overline{V}}{\partial t} + \overline{V}_1 - Da\nabla^2 \overline{V} = -\nabla p + R[1+\delta \sin(\Omega t)]T\bar{k} \tag{5}$$

$$\frac{\partial T}{\partial t} + \overline{V}\cdot\nabla T = \left(\eta \nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)T \tag{6}$$

where $\nabla_H^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian operator. In Eq. (5) $\Omega = \omega_* H_*^2/\lambda_z$ represents the scaled frequency and $\delta = \kappa Fr \Omega^2$ represents the amplitude, where $\kappa = b_*/H_*$ and $Fr = \lambda_{z*}^2/(g_* H_*^3)$, the modified Froude number. The parameter Va is the Vadasz number and is defined as

$$Va = \frac{\phi Pr}{Da} \tag{7}$$

where $Pr = \nu_*/\lambda_{z*}$ is the Prandtl number and $Da = \mu_{e*} K_{z*}/\mu_* H_*^2$ the Darcy number. The boundaries are assumed to be flat and stress-free, that is $w = 0$ and $(\partial u/\partial z) = (\partial v/\partial z) = 0$. The temperature boundary conditions are $T = 1$ at $z = 0$ and $T = 0$ at $z = 1$.

Eqs. (4)–(6) together with the corresponding boundary conditions accept a basic state given by $T_B = 1 - z$ and $V_B = 0$. We assume small perturbations around the basic solution in the form $V = V_B + V'$, $p = p_B + p'$ and $T = T_B + T'$. Substituting these in Eqs. (4)–(6), eliminating the pressure by operating curl twice in Eq. (5) and projecting it on the z-direction we obtain the following equations governing the perturbations as

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}\right)\nabla^2 w' + \nabla_H^2 w' + \frac{1}{\xi}\frac{\partial^2 w'}{\partial z^2} - Da\nabla^2\nabla^2 w' - R[1+\delta\sin(\Omega t)]\nabla_H^2 T' = 0 \tag{8}$$

$$\left[\frac{\partial}{\partial t} - \left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)\right]T' - w' = 0 \tag{9}$$

where w' is the vertical component of the perturbed velocity. The appropriate boundary conditions are $w' = (\partial^2 w'/\partial z^2) = T' = 0$ at $z = 0$ and $z = 1$. Eliminating w' from Eqs. (8) and (9) we obtain a single equation

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}\right)\nabla^2\left[\frac{\partial}{\partial t} - \left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)\right]T' + \nabla_H^2\left[\frac{\partial}{\partial t} - \left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)\right]T' + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\left[\frac{\partial}{\partial t} - \left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)\right]T' - Da\nabla^2\nabla^2\left[\frac{\partial}{\partial t} - \left(\eta\nabla_H^2 + \frac{\partial^2}{\partial z^2}\right)\right] \times T' - R[1+\delta\sin(\Omega t)]\nabla_H^2 T' = 0 \tag{10}$$

We assume a normal mode expansion in the x- and y-direction and time-dependent amplitude $\theta(t)$,

$$T' = \theta(t)\exp\left[i(s_x x + s_y y)\right]\sin(\pi z) \tag{11}$$

where $s^2 = s_x^2 + s_y^2$. Substituting Eq. (11) into the Eq. (10) we arrive at

$$\frac{d^2\theta}{dt^2} + 2p\frac{d\theta}{dt} - F(\alpha)\gamma\left[(\tilde{R} - \tilde{R}_0) + \tilde{R}\delta\sin(\Omega t)\right]\theta = 0, \tag{12}$$

where $2p = \pi^2[\eta\alpha + 1 + \gamma(((\xi\alpha+1)/\xi(\alpha+1)) + Da\,\pi^2(\alpha+1))]$, $F(\alpha) = (\pi^4\alpha/(\alpha+1))$, $\alpha = s^2/\pi^2$, $\gamma = Va/\pi^2$, $\tilde{R} = R/\pi^2$ and $\tilde{R}_0$ is the unmodulated Rayleigh number defined as $\tilde{R}_0 = ((\alpha+1)^2/\alpha)[((\xi\alpha+1)(\eta\alpha+1)/\xi(\alpha+1)^2) + Da\pi^2(\eta\alpha+1)]$. Using the transformation $t = (\pi/2 - 2\tau)/\Omega$, Eq. (12) can be expressed in the canonical form of Mathieu's equation (see McLachlan [13]) as

$$\frac{d^2X}{d\tau^2} + [a - 2q\cos(2\tau)]X = 0 \tag{13}$$

The solution to the above equation is of the form $X = \theta(\tau)e^{\sigma\tau}$ where $\theta(\tau)$ is a periodic function with a period of π or 2π and σ is a characteristic exponent which is a complex number and is a function of a and q. Here the definitions for a, q and σ are obtained upon transforming Eq. (12) to the canonical form and are given by

$$\frac{2}{\sqrt{-a}} = \frac{\Omega}{\left[F(\alpha)\gamma(\tilde{R} - \zeta)\right]^{1/2}} \tag{14}$$

$$\frac{1}{2}q = \frac{F(\alpha)\gamma\tilde{R}\delta}{\Omega^2} = F(\alpha)\gamma\tilde{R}\kappa Fr \tag{15}$$

$$\sigma = -2p/\Omega \tag{16}$$

where ζ is a parameter defined as

$$\zeta = -\tilde{R}_0\frac{\left(\eta\alpha + 1 - \gamma\left(\frac{(\xi\alpha+1)}{\xi(\alpha+1)} + Da\pi^2(\alpha+1)\right)\right)^2}{4\gamma(\alpha+1)\left(\frac{(\xi\alpha+1)(\eta\alpha+1)}{\xi(\alpha+1)^2} + Da\pi^2(\alpha+1)\right)} \tag{17}$$

3. Solution procedure

The characteristic exponent with the largest $Re(\sigma)$ determines the stability of the system. The disturbances amplify if $Re(\sigma) > 0$ and decay if $Re(\sigma) < 0$. If $Re(\sigma) = 0$, the solution to Eq. (13) is defined in terms of Mathieu's functions $ce_n(a,q)$ and $se_m(a,q)$, $n = 0,1,2,\ldots$, $m = 1,2,\ldots$ (see [13]). Fig. 1(a) shows the plots of ce_0, se_1 and ce_1 which separate the stable and unstable solutions. If the other Mathieu's functions for $n,m > 1$ are superimposed in Fig. 1(a) one would observe that the regions separated by Mathieu's functions in the a–q plane are alternately stable and unstable. For our analysis we consider only small values of q and hence the analysis around the lower order functions ce_0, se_1 and ce_1 is sufficient. The region below curve ce_0 and the region enclosed between curves se_1 and ce_1 correspond to the unstable zones. The narrow region between curves ce_0 and se_1 represents the stable zone. The regions enclosed by ce_n with even indices (i.e. $n = 0,2,4,6,\ldots$) yield synchronous solutions and those enclosed by ce_n with odd indices (i.e. $n = 1,3,5,7,\ldots$) yield subharmonic solutions thus implying that the a–q plane consists of alternating regions of synchronous and subharmonic solutions.

Suppose $\{y_1, y_2\}$ constitute a fundamental system of Eq. (13). We used the relation $\cosh(\sigma\pi) = y_1(\pi)$ and the conditions $y_1(0) = 1, y_1'(0) = 0, y_2(0) = 0, y_2'(0) = 1$ in the case of synchronous solutions and the relation $\cosh(2\sigma\pi) = (y_1(\pi) + y_2'(\pi))/2$ and the conditions $y_1(-\pi) = y_2'(-\pi) = 1, y_2(-\pi) = y_1'(-\pi) = 0$ in the case of subharmonic solutions to calculate σ. There are solutions to Eq. (13) for $a < 0$ also and q may be replaced by $-q$ with no effect on the solution.

For a porous medium heated from below, an inverted pendulum analogue, the numerical values for a are less than zero and are defined by Eq. (14). A chart of $q/2$ against $2/\sqrt{-a}$, for various values

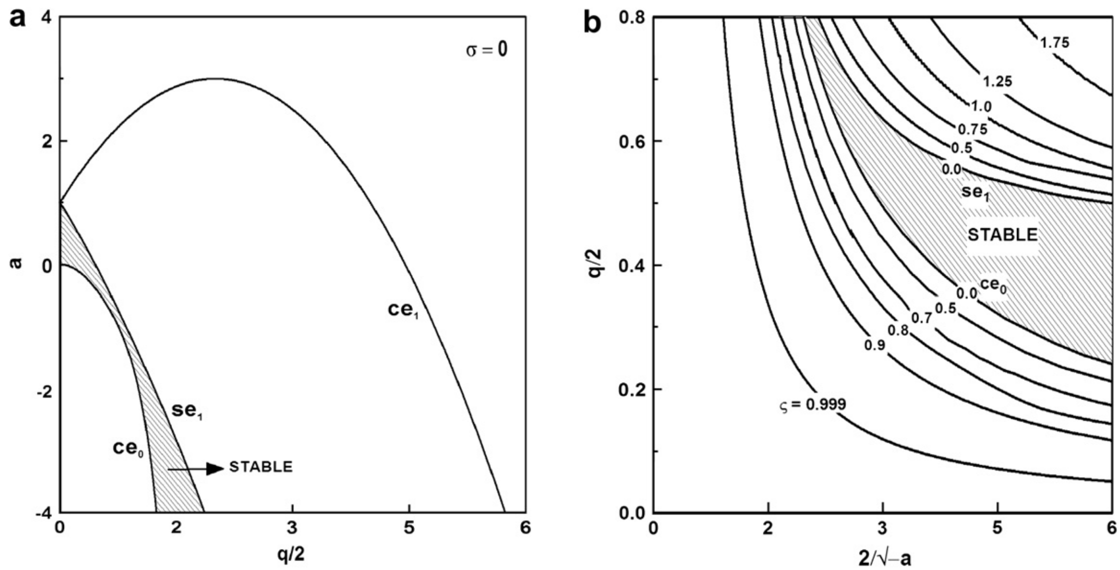

Fig. 1. Stable regions as a function of *a* and *q*. (a) Lower order Mathieu's functions depicting stable and unstable regions, (b) stability chart for Mathieu's equation for various values of ς.

of the modified characteristic exponent $\varsigma = \sigma/\sqrt{-a}$, is shown in Fig. 1(b) for small values of *q*. In Fig. 1(b), $\varsigma = 0$ refers to Mathieu's functions depicted by the curves for ce_0 and se_1. We may now present a relation for the characteristic Rayleigh number in terms of the newly defined parameter ς, by substituting $\varsigma = \sigma/\sqrt{-a}$ in Eq. (14),

$$\tilde{R} = \zeta + \frac{(\tilde{R}_0 - \zeta)}{\varsigma^2} \tag{18}$$

Fig. 1(b) together with Eqs. (14)–(18) may be used to evaluate the critical Rayleigh number and wavenumbers in terms of the frequency Ω, the parameters (κFr) and γ. We evaluated the characteristic Rayleigh number versus the frequency for selected values of α as follows. For a selected value of ς we evaluated $\tilde{R}$ using Eq. (18) and the value for *q*/2 using Eq. (15). Then Fig. 1(b) was used to find the corresponding value of $2/\sqrt{-a}$ and then the frequency was calculated from Eq. (14). The critical Rayleigh number R_{cr} is then obtained by minimizing R_c over α.

4. Results and discussion

The effect of gravity modulation on the onset of buoyancy convection in a fluid saturated anisotropic porous medium is studied using Brinkman's equation. Based on the previous works and available data we considered *Da* in the range $10^{-4} \le Da \le 10^{-1}$ and ξ and η in the range $10^{-1} \le \xi, \eta \le 10$. Following [8] we fixed $(\kappa Fr) = O(10^{-5})$ corresponding to the solidification of binary liquid metals and $\gamma = O(3)$. We observed that that the stability characteristics remain unaffected for sufficiently higher frequencies consistent with earlier results available [6,10].

Before proceeding further it is of interest to compare the results corresponding to an isotropic porous medium of Darcian nature $(Da \to 0, \xi = \eta = 1)$, a particular case of the present problem, with

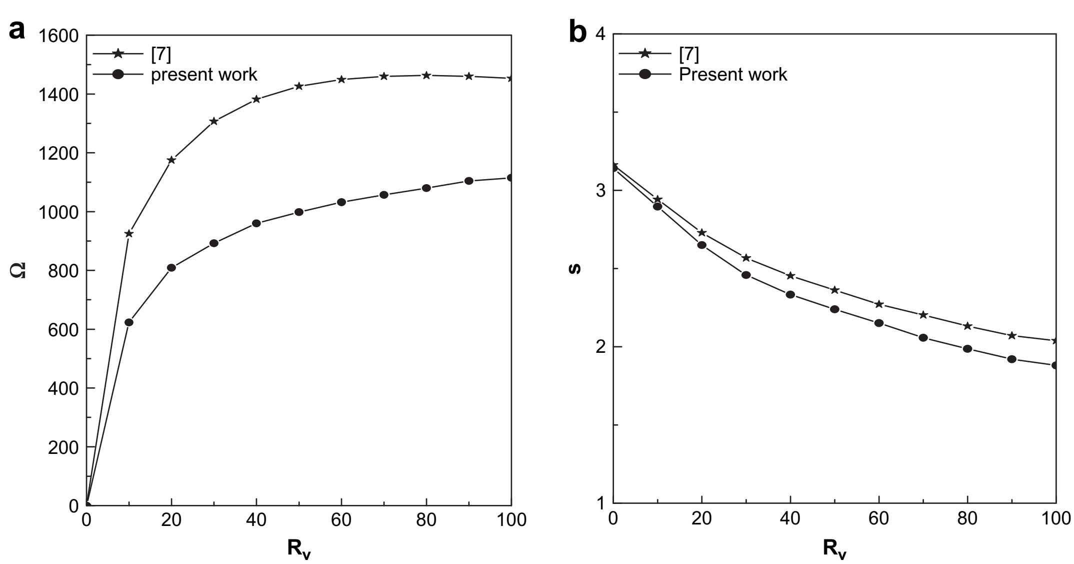

Fig. 2. Comparison of the present results with those of [7]. (a) Frequency against vibrational Rayleigh number, (b) wavenumber against vibrational Rayleigh number.

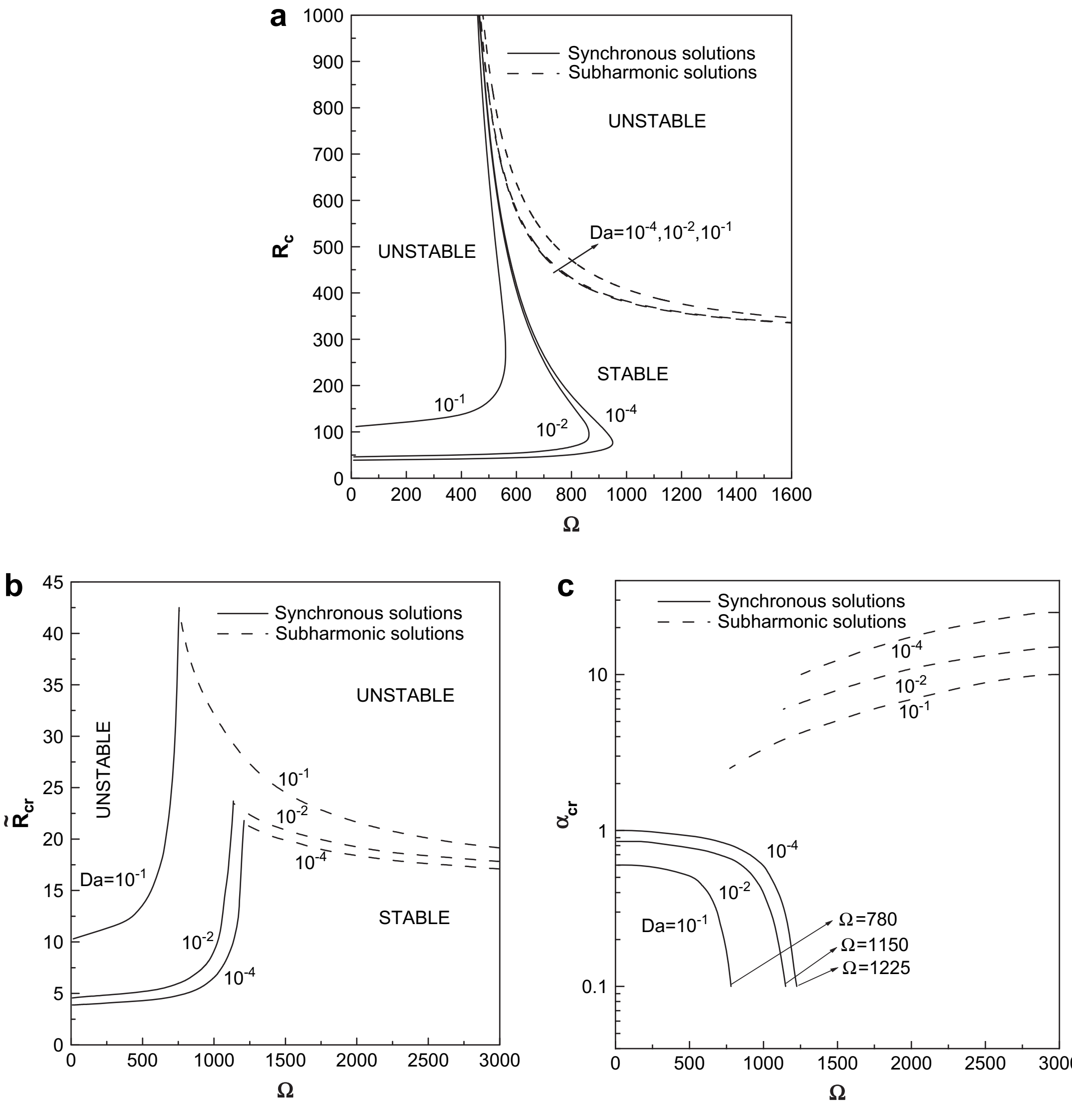

Fig. 3. (a) Characteristic Rayleigh number against frequency for $\alpha = 1$, $\xi = 1$, $\eta = 1$ and different values of *Da*. (b) Critical Rayleigh number against frequency for $\xi = 1$, $\eta = 1$ and different values of *Da*. (c) Critical wavenumber against frequency for $\xi = 1$, $\eta = 1$ and different values of *Da*.

the existing results. In the unmodulated limit $\Omega \to 0$, the critical Rayleigh number R_{cr} and critical wavenumber α_{cr} approach the values $3.9272\pi^2$ and 1 against the known exact values $4\pi^2$ and 1 (see [8,9]) respectively. In the presence of modulation the transition frequency separating the synchronous and subharmonic modes occurs at $\Omega = 1219.3548$ against $\Omega \cong 1225$ of [8,9]. Fig. 2 exhibits a comparison with the limiting case $A_L \to \infty$, corresponding to an infinite horizontal layer, of Barden and Mojtabi [7] for different values of their vibrational Rayleigh number R_v which is proportional to b_*^2. We observe that the agreement between the two results is qualitatively correct but yet in the case of frequency it is quantitatively approximate which may be anticipated due to the time-averaging procedure employed in [7]. We however note that sufficiently higher values of R_v result in larger modulation amplitudes and hence the validity of the time-averaging method becomes questionable.

The characteristic Rayleigh number R_c is plotted against the frequency Ω for various values of *Da* when $\alpha = 1.0$ and is shown in Fig. 3(a). It corresponds to the case of an isotropic porous medium ($\xi = \eta = 1$). It is noted that the region below the curves is stable and the region above them is unstable for both synchronous and subharmonic modes. It is noted that the synchronous mode is much affected by changes in the values of *Da* compared to the subharmonic one. The critical Rayleigh number $\tilde{R}_{cr}$ and critical wavenumber α_{cr} against frequency Ω are plotted in Fig. 3(b) and (c) for different values of *Da*. We note that the limit $\Omega \to 0$ corresponds to the unmodulated case, i.e., a porous medium subjected to a constant gravitational field. It is clear that the vibration frequency inhibits the onset of convection in the region of synchronous response whereas augments it in the region of subharmonic response for all values of Da. In the synchronous mode, the inhibition is weak for lower frequencies and becomes strong for higher frequencies until Ω reaches a transition value beyond which subharmonic mode becomes critical. The transitions occur at $\Omega = 1225,\ 1150$ and 780 for $Da = 10^{-4},\ 10^{-2}$and 10^{-1} respectively, i.e., the transition point gets shifted to lower frequencies as the porous medium becomes sparse. $\tilde{R}_{cr}$ decreases in the region of subharmonic response and becomes invariant for sufficiently large values of Ω. The corresponding α_{cr} decreases for the synchronous mode and increases for the subharmonic mode with a sudden change in it at the point of transition as the vibration frequency is increased. This shows that the heat transfer

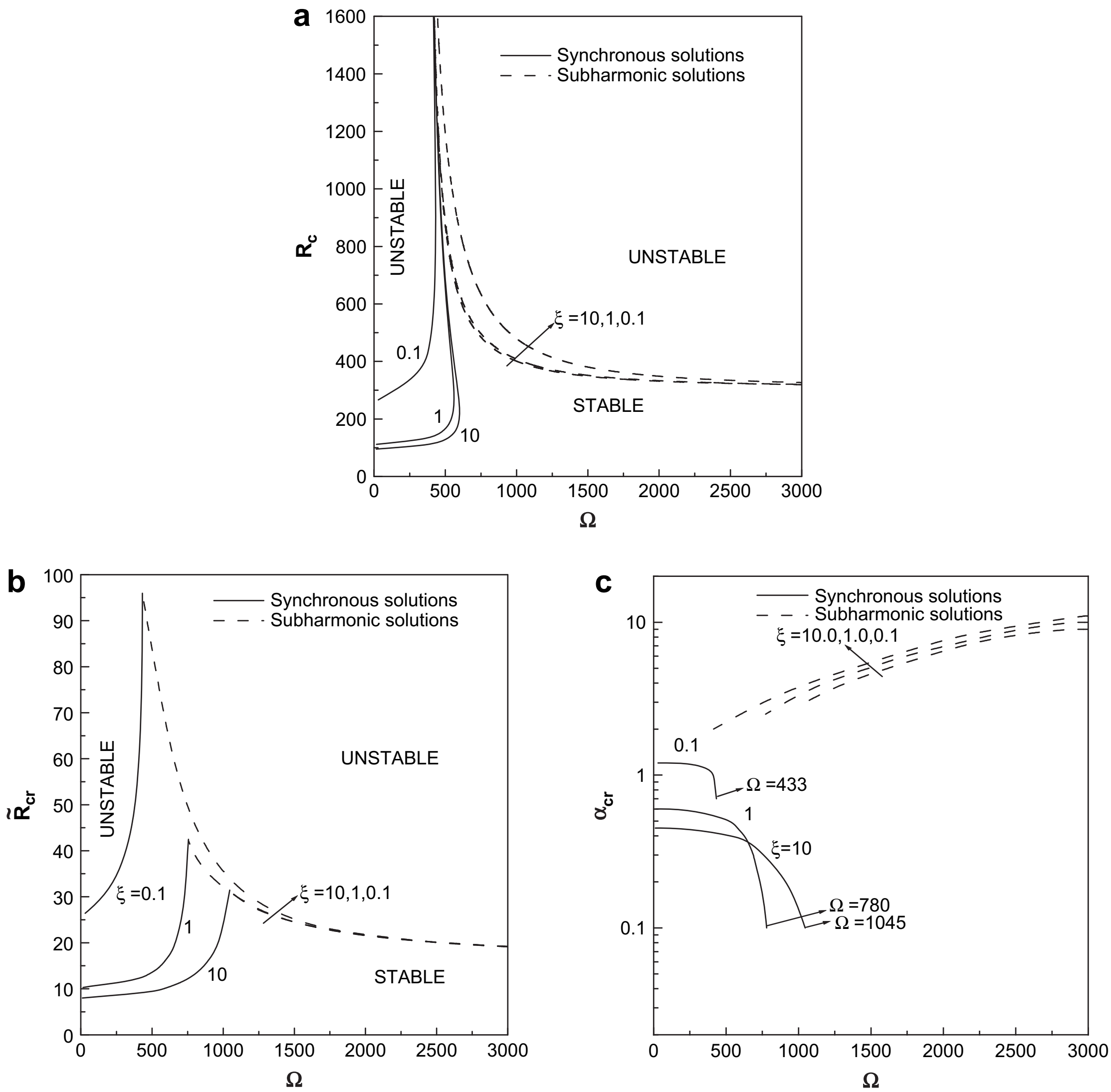

Fig. 4. (a) Characteristic Rayleigh number against frequency for $\alpha = 1$, $Da = 10^{-1}$, $\eta = 1$ and different values of ξ. (b) Critical Rayleigh number against frequency for $Da = 10^{-1}$, $\eta = 1$ and different values of ξ. (c) Critical wavenumber against frequency for $Da = 10^{-1}$, $\eta = 1$ and different values of ξ.

characteristics near the point of transition can be well controlled by a suitable choice of Da. Also we observe that $\tilde{R}_{cr}$ decreases drastically and the convection cells are suppressed as $Da \to 0$ consistent with the physical reasoning.

Fig. 4 shows the stability characteristics against the vibration frequency Ω for various values of ξ when $Da = 10^{-1}$. $\tilde{R}_{cr}$ plotted against Ω in Fig. 4(b) shows that that eventhough an increase in ξ reduces R_c in both synchronous and subharmonic modes its effect is prominent only in the synchronous mode. The synchronous mode of instability changes to the subharmonic one when Ω becomes 433, 780 and 1045 respectively for $\xi = 0.1$, 1 and 10. Thus ξ shifts the transition point to a higher frequency region. The corresponding α_{cr} plotted against Ω in Fig. 4(c) shows strengthening of the circulation patterns for an increase in ξ. Moreover we observe that the critical wavenumber decreases with increasing frequencies for the synchronous mode and increases for the subharmonic mode. The jump in α_{cr} near the point of transition is found to be a strong function of ξ. Similarly the stability characteristics for various values of η when $Da = 10^{-1}$ is displayed in Fig. 5. It is evident from Fig. 5(a) that when η increases, R_c also increases for both synchronous and subharmonic modes, favouring the quiescent base state. The critical Rayleigh number $\tilde{R}_{cr}$ and critical wavenumber α_{cr} against frequency Ω are shown in Fig. 5(b) and (c) for different values of η. The effect of η is found to stabilize the diffusive solution and delay the onset of convection. We see that anisotropy in conductivity affects the subharmonic mode as well significantly compared to that in permeability. The transition between the two destabilizing modes occurs at $\Omega = 802, 780$ and 632 for $\eta = 0.1,\ 1$ and 10 respectively. We observe that the amplitude of the disturbance pattern is reduced considerably with increasing η for both synchronous and subharmonic solutions.

5. Conclusion

We investigated the effect of gravity modulation on the onset of buoyancy driven convection in a horizontal porous layer. The porous medium was assumed to be anisotropic and governed by Brinkman's equation. It leads to the following conclusions. The instability mode changes from synchronous to subharmonic as the vibration frequency increases to a certain level. The synchronous mode is affected much by the non-Darcian effects as well as anisotropies of the porous matrix. The mechanical and thermal

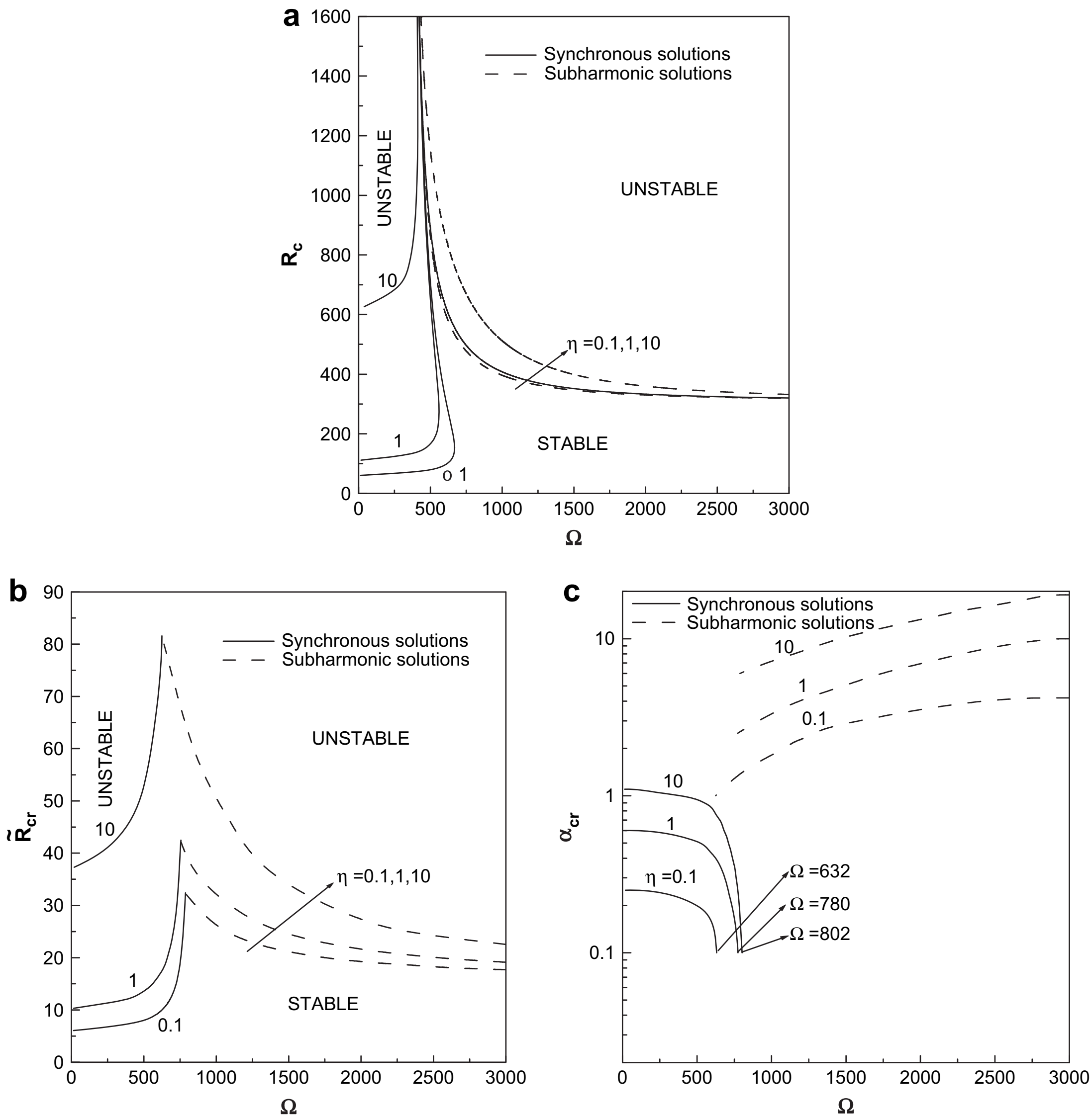

Fig. 5. (a) Characteristic Rayleigh number against frequency for $\alpha = 1$, $Da = 10^{-1}$, $\xi = 1$ and different values of η. (b) Critical Rayleigh number against frequency for $Da = 10^{-1}$, $\xi = 1$ and different values of η. (c) Critical wavenumber against frequency for $Da = 10^{-1}$, $\xi = 1$ and different values of η.

anisotropies produce opposite effects on the onset of motion and its secondary characteristics. The transitional frequency gets shifted to a lower frequency range when either the medium becomes sparse or the thermal anisotropy parameter becomes large and to a higher frequency range when the mechanical anisotropy parameter becomes large.

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